THE DISK SETTLING INSTABILITY
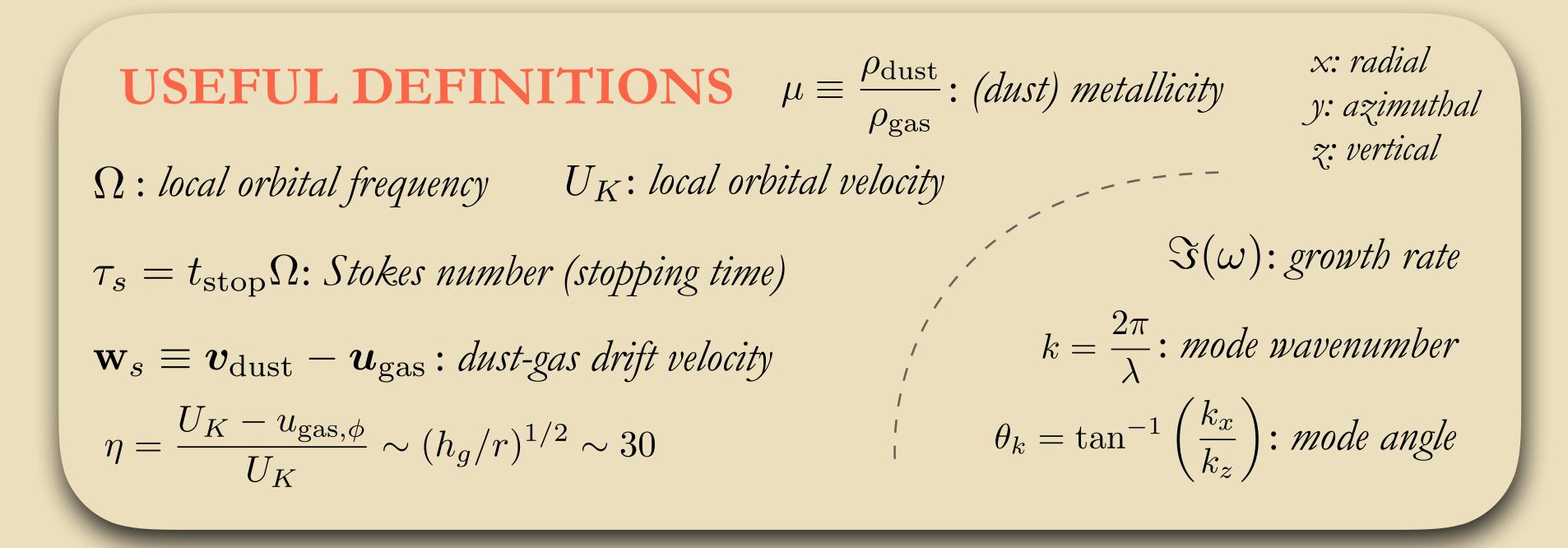
Fast clumping of small grains in protoplanetary disks

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ABSTRACT

We identify and characterize the disk "settling instability," a variant of the streaming instability (Youdin & Goodman 2005) that can occur in protoplanetary disks when grains settle vertically into the midplane region. The growth rate of the settling instability is independent of grain size and comparable to the disk orbital frequency for realistic parameters, suggesting that the instability is relevant for even the smallest grains. In addition, its characteristic wavelengths are larger than those of the streaming instability, potentially allowing the instability to concentrate large masses of solids. These properties suggest that in the process of settling, small grains could band into rings (then filaments or clumps) with significantly higher than background metallicity. This could have a variety of important consequences for the early stages of planetesimal formation, for instance, by enhancing coagulation rates of small grains and/or creating high-metallicity regions in the midplane that act to seed the standard streaming instability.
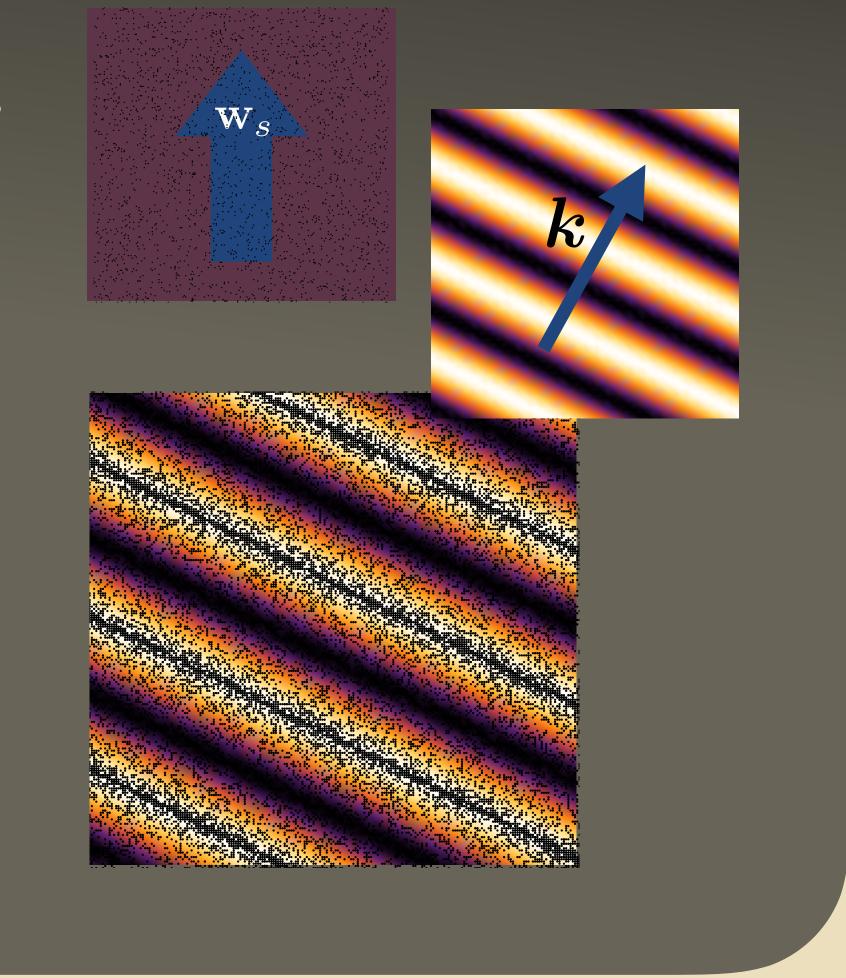
USEFUL DEFINITIONS

$\mu \equiv \dfrac{\rho_{\rm dust}}{\rho_{\rm gas}}$: (dust) metallicity

Ω : local orbital frequency $\quad U_K$: local orbital velocity

$\tau_s = t_{\rm stop}\Omega$: Stokes number (stopping time)

$\mathbf{w}_s \equiv \boldsymbol{v}_{\rm dust} - \boldsymbol{u}_{\rm gas}$: dust-gas drift velocity

$\eta = \dfrac{U_K - u_{{\rm gas},\phi}}{U_K} \sim (h_g/r)^{1/2} \sim 30$

x: radial
y: azimuthal
z: vertical

$\Im(\omega)$: growth rate

$k = \dfrac{2\pi}{\lambda}$: mode wavenumber

$\theta_k = \tan^{-1}\left(\dfrac{k_x}{k_z}\right)$: mode angle

RESONANT DRAG INSTABILITY arXiv:1706.05020

An algorithm to find instabilities in coupled dust dust-gas mixtures

1) Work out equilibrium: drift velocity ($\mathbf{w}_s$), gas properties

2) Choose a gas wave, with frequency $\omega_{\rm wave}(\boldsymbol{k})$

   Sound wave   MHD wave   Epicycles   Gravity wave

3) Match the dust frequency $\boldsymbol{k}\cdot\mathbf{w}_s$ to $\omega_{\rm wave}(\boldsymbol{k})$ gives "resonant wavenumber" $k_{\rm res}$

4) Dust "sees" a stationary wave, clumps dust

   dust backreaction on gas   drag forces on dust

   Growth rate easy to calculate $\approx \mu^{1/2}\left[(\xi_{\mathcal{F}}^L T_{\rho_d}^{(1)})\,(\boldsymbol{k}^T D_{\rm drag}^{-1} C_{\boldsymbol{v}}\xi_{\mathcal{F}}^R)\right]^{1/2}$

   gas eigenmode

COUPLED GAS+DUST EQUATIONS IN A LOCAL PATCH OF DISK

STREAMING INSTABILITY

Dust in midplane, streams through gas with velocity

$\mathbf{w}_s = \boldsymbol{v}_{\rm dust} - \boldsymbol{u}_{\rm gas} = -2\dfrac{\eta U_K(1+\mu)\tau_s}{(1+\mu)^2+\tau_s^2}\hat{\boldsymbol{x}} + \dfrac{\eta U_K\tau_s^2}{(1+\mu)^2+\tau_s^2}\hat{\boldsymbol{y}}$

$\approx -2\eta U_K\tau_s\hat{\boldsymbol{x}}, \quad \mu \ll 1, \; \tau_s \ll 1$ (Nakagawa-Sekiya-Hayashi drift)

Gas epicycles have frequency $\omega_{\rm gas} = \pm\dfrac{k_z}{k}\Omega$

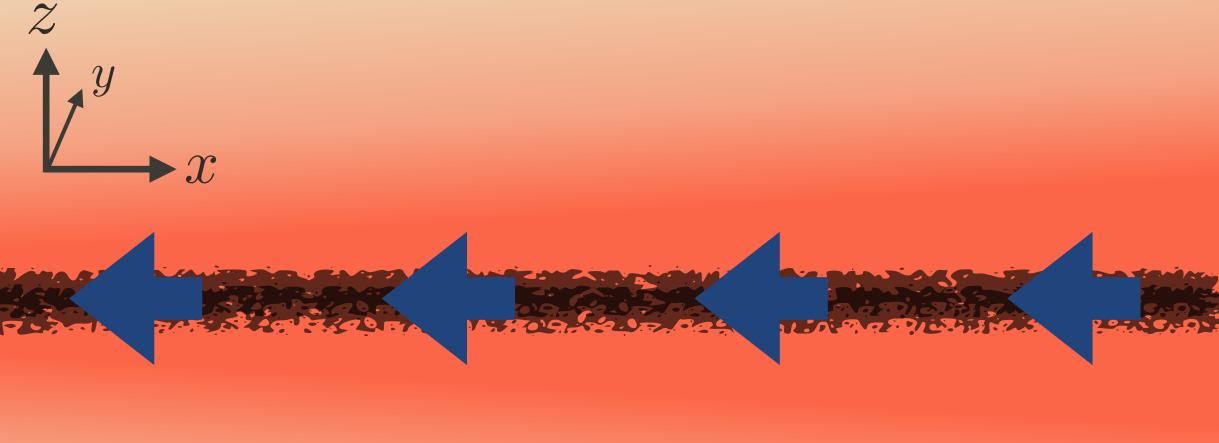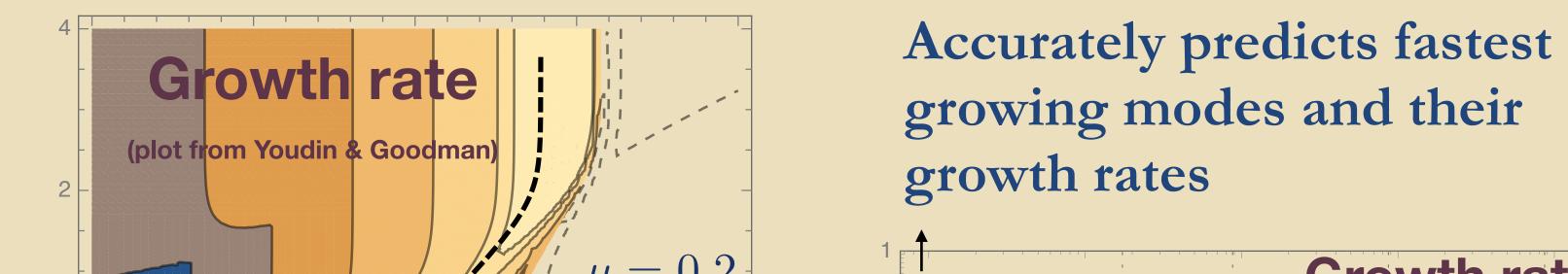**Resonance (fastest growing mode) occurs at** $k_{\rm res} = \dfrac{\Omega}{|{\rm w}_{s,x}\tan\theta_k|}$

Growth rate is $\Im(\omega) \approx \Omega\tau_s\sqrt{\dfrac{\mu}{8}(2\hat{k}_z^2+\hat{k}_x^2)}$
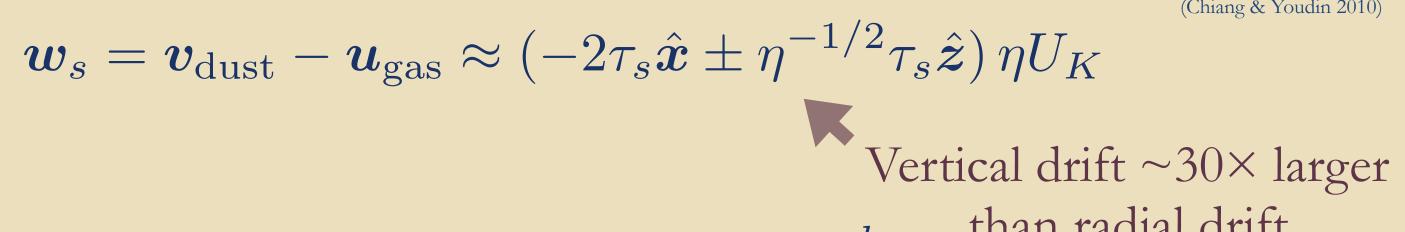
Occurs because the streaming dust excites gas epicycles

Growth rate (plot from Youdin & Goodman)

$\log_{10}(k_z\eta r)$ $\mu = 0.2$ $\tau_s = 0.01$

resonance $\qquad k_{\rm res} = \dfrac{\Omega}{|{\rm w}_{s,x}\tan\theta_k|}$

long wavelengths $\log_{10}(k_x\eta r)$

Accurately predicts fastest growing modes and their growth rates

Growth rate $\theta_k = 30°$

$\tau_s = 1$ $\quad \tau_s = 0.1 \quad \tau_s = 0.01 \quad \tau_s = 0.001$

$\mu = 0.001$

long wavelengths $k\eta r$

At $\mu > 1$ a different mode dominates (see appendix of paper)

Growth rate $\omega \approx i\sqrt{\mu-1}$, slower (but larger scales) for large grains

SETTLING INSTABILITY

Dust also settles vertically if spread throughout disk (Chiang & Youdin 2010)

$\boldsymbol{w}_s = \boldsymbol{v}_{\rm dust} - \boldsymbol{u}_{\rm gas} \approx (-2\tau_s\hat{\boldsymbol{x}} \pm \eta^{-1/2}\tau_s\hat{\boldsymbol{z}})\,\eta U_K$

Vertical drift ~30× larger than radial drift

Resonates with gas epicycles $\omega_{\rm gas} = \pm\dfrac{k_z}{k}\Omega$

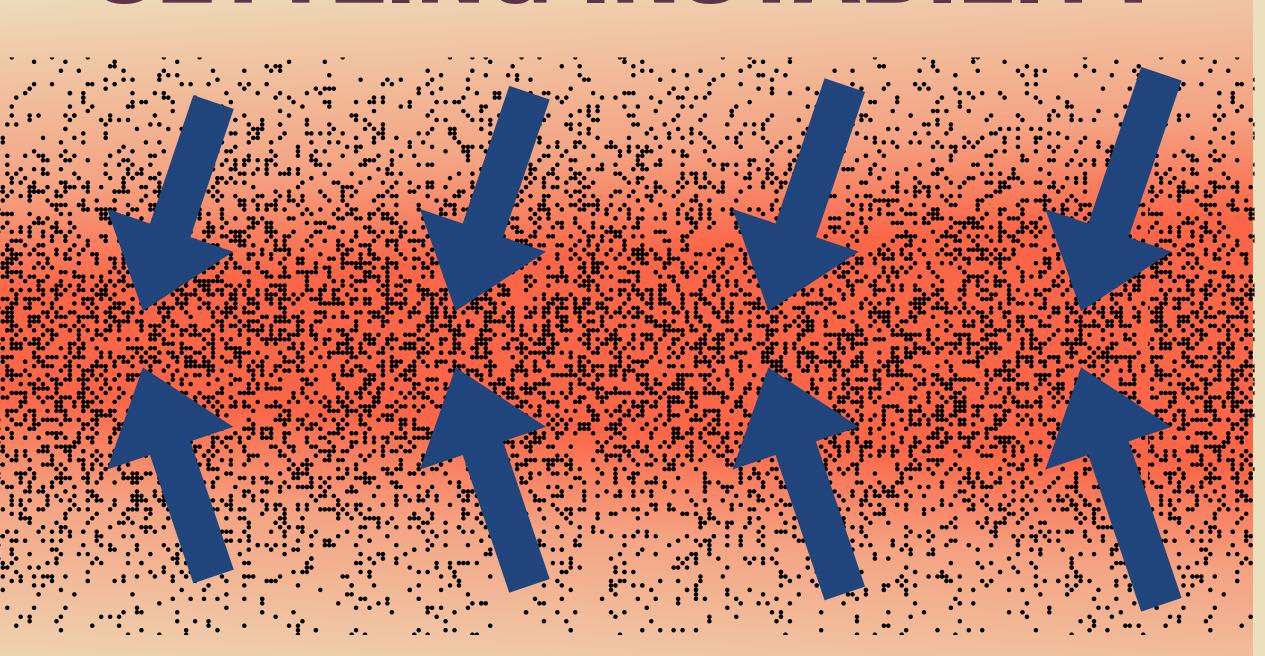**Resonance (fastest growing mode) occurs at**
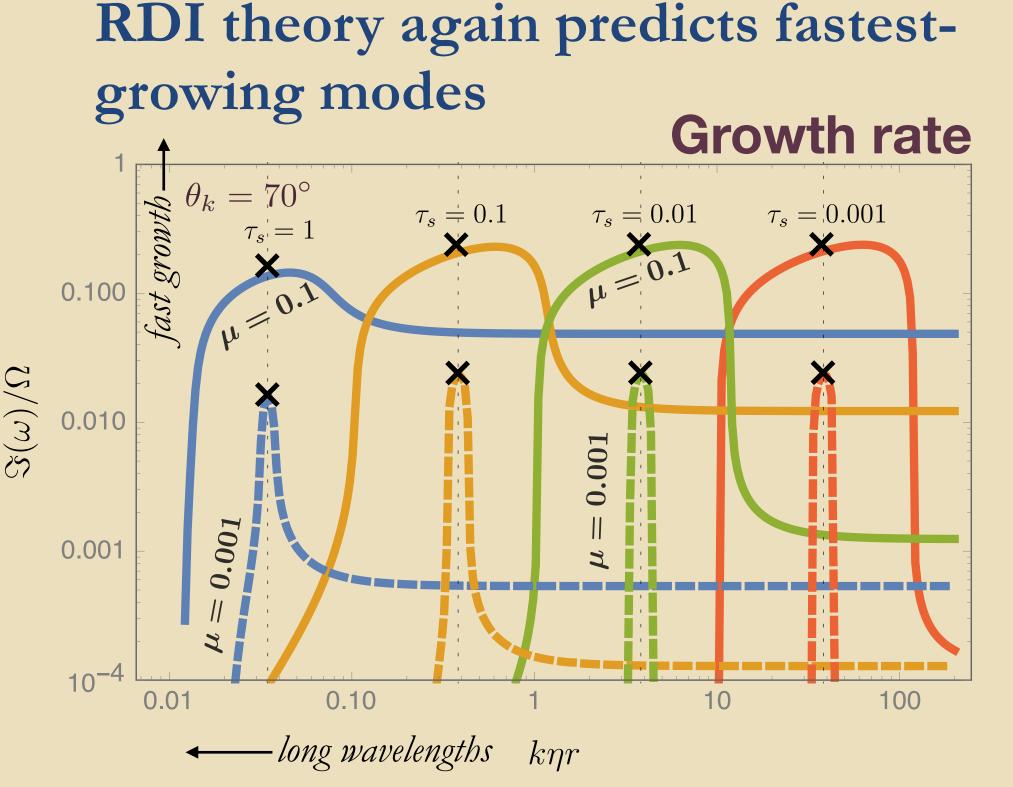
$k_{\rm res} = \dfrac{\Omega}{|\tan\theta_k{\rm w}_{s,x}+{\rm w}_{s,z}|} \approx \dfrac{\Omega}{|{\rm w}_{s,z}|}$ ~30× larger scale than streaming instability

Growth rate is

$\Im(\omega) \approx \Omega\left(\dfrac{\mu}{2}\right)^{1/2}|\sin\theta_k|$ Growth rate does not depend on grain size!
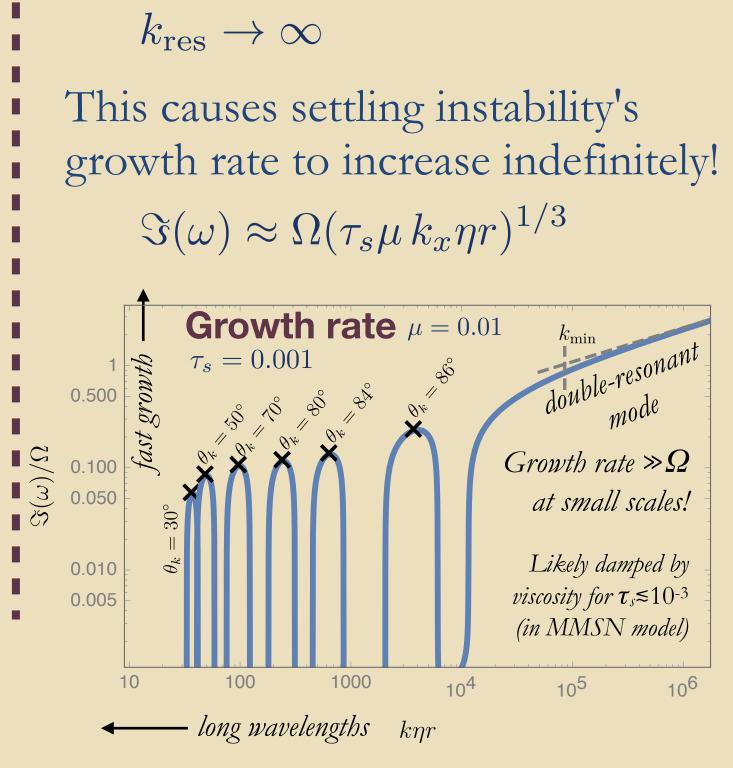
RDI theory again predicts fastest-growing modes

Growth rate

$\theta_k = 70°$ $\tau_s = 1$ $\quad \tau_s = 0.1 \quad \tau_s = 0.01 \quad \tau_s = 0.001$

$\mu = 0.1$ $\quad \mu = 0.1$

$\mu = 0.001$ $\quad \mu = 0.001$

long wavelengths $k\eta r$

Weak dependence on metallicity (~$\mu^{1/2}$)
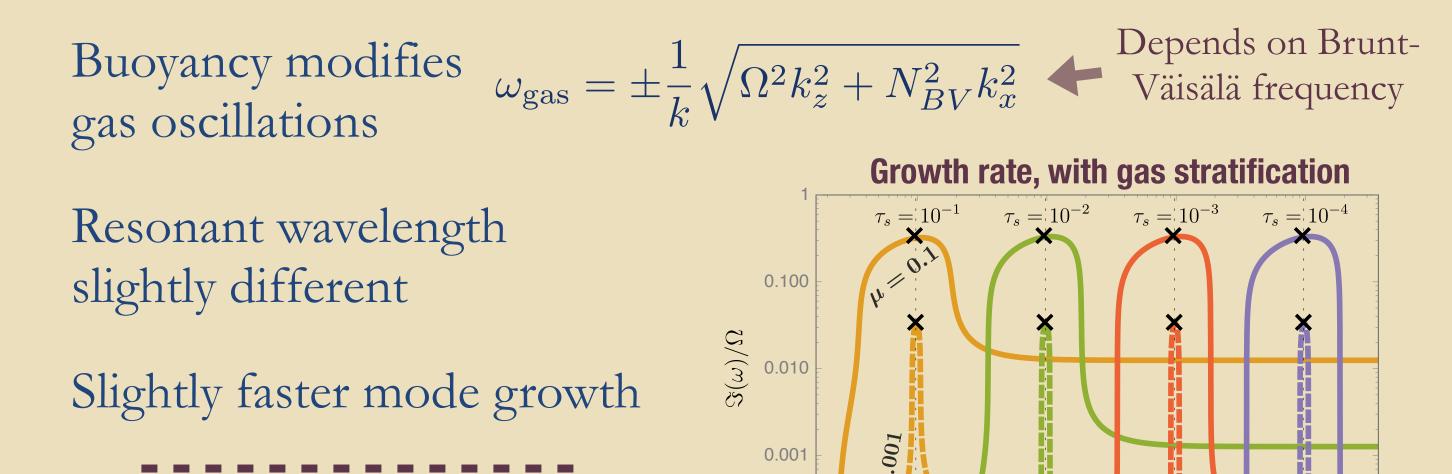
"Double-resonant mode"

For mode angle $\tan\theta_k = {\rm w}_{s,z}/{\rm w}_{s,x}$

$k_{\rm res} \to \infty$

This causes settling instability's growth rate to increase indefinitely!

$\Im(\omega) \approx \Omega(\tau_s\mu\,k_x\eta r)^{1/3}$

Growth rate $\tau_s = 0.001$ $\mu = 0.01$

double-resonant mode

Growth rate $\gg \Omega$ at small scales!

Likely damped by viscosity for $\tau_s \approx 10^{-3}$ (in MMSN model)

long wavelengths $k\eta r$

Possible IMPLICATIONS

For smaller grains, growth time much shorter than settling time

$$t_{\rm settle} \sim (\Omega\tau_s)^{-1}$$

Minor modifications from gas stratification see also Lambrechts+ (2016)

Buoyancy modifies gas oscillations $\omega_{\rm gas} = \pm\dfrac{1}{k}\sqrt{\Omega^2 k_z^2 + N_{BV}^2 k_x^2}$ Depends on Brunt-Väisälä frequency

Resonant wavelength slightly different

Slightly faster mode growth

Growth rate, with gas stratification

$\tau_s = 10^{-1}, \quad \tau_s = 10^{-2}, \quad \tau_s = 10^{-3}, \quad \tau_s = 10^{-4}$

$\mu = 0.001$

$k\eta r$

Grows faster than settling for

$\tau_s \lesssim 0.1\left(\dfrac{\mu}{0.01}\right)^{1/2}$

Grains sizes not affected by gas viscosity

$R_d \gtrsim 8\left(\dfrac{r}{\rm AU}\right)^{-3/4}\mu{\rm m}$

May be affected by turbulence for smaller grains

$\tau_s \lesssim 0.01\left(\dfrac{\alpha}{10^{-4}}\right)^{1/2}\left(\dfrac{\mu}{0.01}\right)^{-3/4}$

(but generally less so than YG streaming instability)

Enough mass in radial wavelength for planetesimal of size

$R \sim 1000{\rm km}\left(\dfrac{\tau_s}{0.001}\right)^{1/3}\left(\dfrac{\mu}{0.01}\right)^{1/3}\left(\dfrac{r}{\rm AU}\right)^{-0.26}$

Initial small grain population evenly spread through disk

Instability grows fastest for near-radial modes

AS THE DUST SETTLES

Helps grain growth during settling?
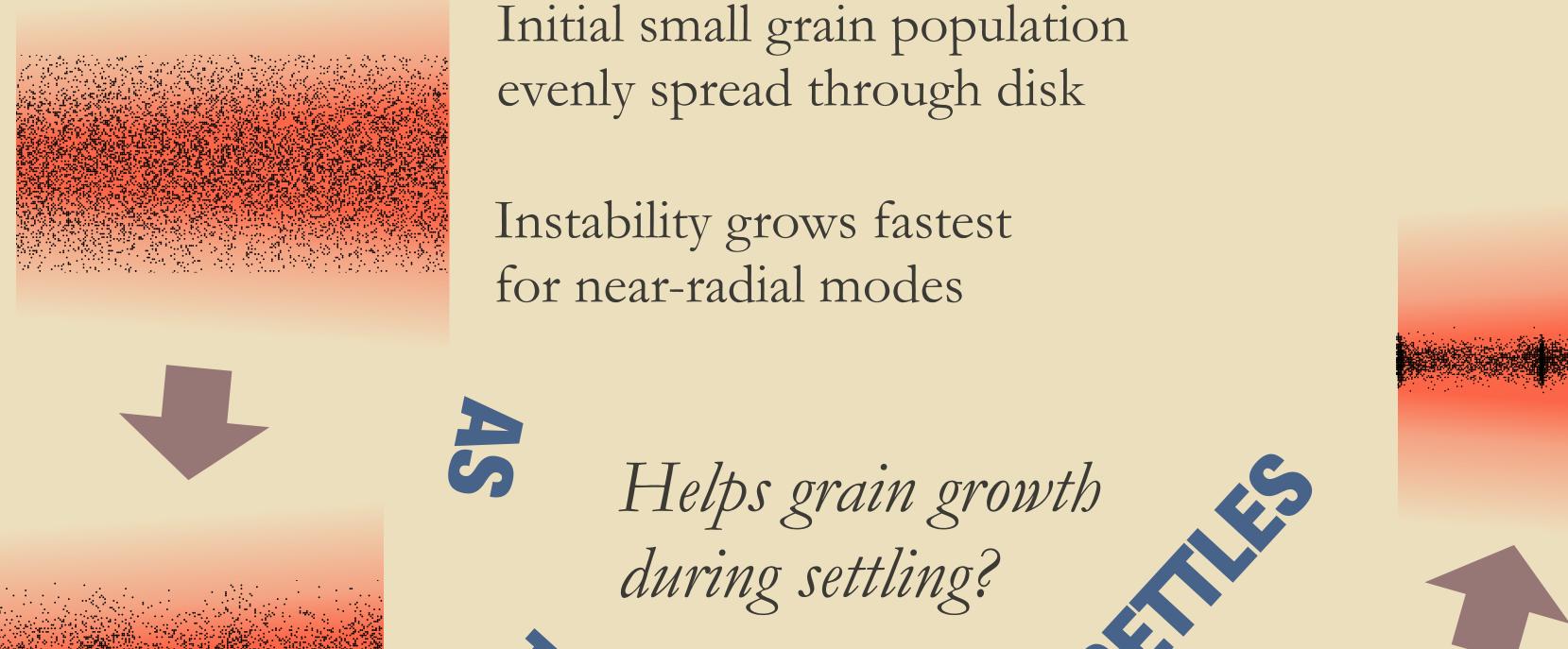
Causes local density enhancements

Clumps contain a large mass of solids

Enhances coagulation rate in clumps during settling? Dzglkowska & Dullemond (2014)

Would naturally produce high-metallicity regions in midplane

Simulations show a critical metallicity $Z \gtrsim 3Z_{MMSN}$ for the streaming instability to form planetesimals (Johansen+ 2009, Bai & Stone 2010, Carrera+ 2015)

Does the disk settling instability provide high-Z seed regions for the YG streaming instability?

Or enhanced densities could seed pressure bumps?

Or even direct collapse of high-density regions?

Numerical simulations needed...